

Prediction of Discretization Error using Error Transport Equation

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Objective

- Develop a robust discretization error field prediction algorithm for a selected transport variable utilizing an error transport equation preferably on a single mesh or at most two geometrically similar meshes.

Introduction

Why Quantify discretization Errors?

- Industrial design and analysis requires quantification of all uncertainties in simulations as well as experimental uncertainty
- Discretization and modeling errors are dominant contributors to the overall simulation uncertainty
- Modeling error can not be assessed unless discretization error is quantified first.
- Quantification of discretization errors may guide grid refinement strategies to reduce it further an hence achieving greater accuracy

Theory

- Let $\phi(x, t)$ is a dependent variable governed by an equation with differential operator L , and the discrete operator L_h

$$L(\phi) = 0 \quad (1)$$

$$L_h(\phi_h) = 0 \quad (2)$$

ϕ_h denotes the numerical solution on a mesh represented by mesh size 'h'.

Let ϕ_{hc} be the continuous function obtained by mapping of ϕ_h ; then

$$L(\phi_{hc}) = \tau_h \quad (3)$$

Subtracting (3) from (1) yields the error transport equation (ETE). Note: non-linear terms will yield additional error source terms. If curve fitting is done in space, time discretization errors need to be accounted separately. Taylor series expansion of the time derivative yield the error source term due to time discretization as

$$\tau_t = \left\{ dt * \frac{\phi_{tt}}{2} - dt^2 * \frac{\phi_{ttt}}{6} + Order(dt^3) \right\} * \Delta vol \quad (4)$$

The resulting ETE is

$$L(\Delta\phi) = \tau_h + \tau_t \quad (5)$$

Eq. 5 can be discretized and solved using the schemes available in the original code. For more details see Celik et al. (2016).

Verification: Sol. to non-linear Burger's equation

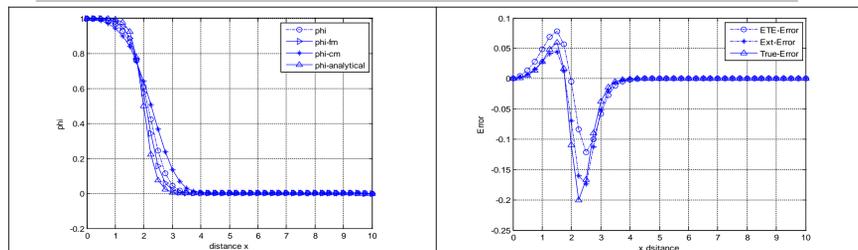


Figure 1. (a) Analytical and numerical solutions and (b) errors, at $t=4.0$ sec, where fm represents fine mesh, cm represent coarse mesh, $U=1.0$, $\gamma = 0.1$, $\Delta t = 0.1$, and $N_x = 41$.

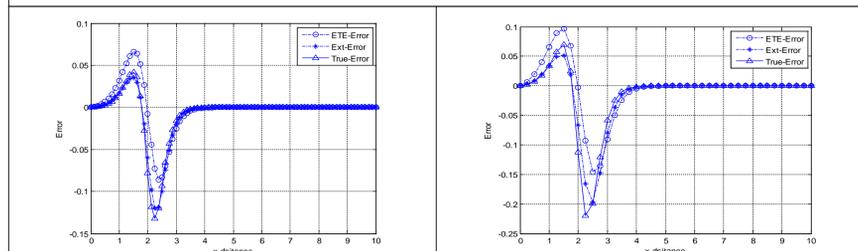


Figure 2. Error at $t=4.0$ sec, $U=1.0$, $\gamma = 0.1$, $\Delta t = 0.1$, and $N_x = 81$. Figure 3. Error at $t=4.0$ sec, $U=1.0$, $\gamma = 0.1$, $\Delta t = 0.25$, and $N_x = 41$.

MFIX Application

Governing equations for MFIx can be found in Benyahia et al. (2007).

ETE for gas-phase volume fraction (Celik et al., 2016):

$$\frac{\partial}{\partial t} (\rho_g \Delta \epsilon_g) + \frac{\partial}{\partial x_j} (\bar{\rho}_g \Delta \epsilon_g \bar{U}_{gj}) = -\tau - \frac{\partial}{\partial x_j} (\bar{\rho}_g \bar{\epsilon}_g \Delta U_{gj}) - \frac{\partial}{\partial x_j} (\bar{\rho}_g \Delta \epsilon_g \Delta U_{gj}) \quad (6)$$

Gas-velocity error is estimated (Celik et al, 2012, 2016) from:

$$E_v^h = \frac{E_a^h}{(1-r^p) \left[1 + \frac{r^p}{(1+r^p)} \right]} \quad ; \quad E_a = U_{gf} - U_{gc} = \Delta U_{gj} \quad (7a,b)$$

The case considered is 1D 'transient core annular flow' problem presented by Benyahia et al (2007)

Results

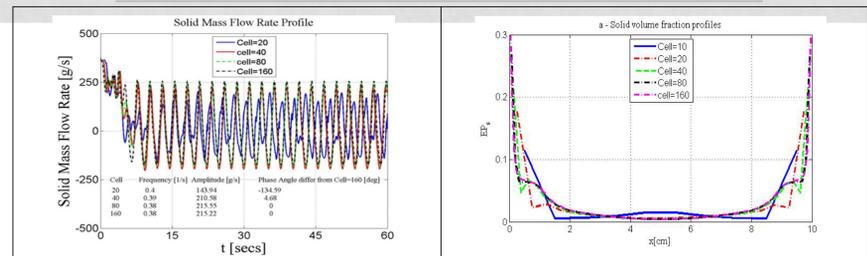


Figure 4. Variation of solid mass flow rate with different mesh size vs. time.

Figure 5. Time averaged quantities for 10, 20, 40, 80, and 160 cell cases.

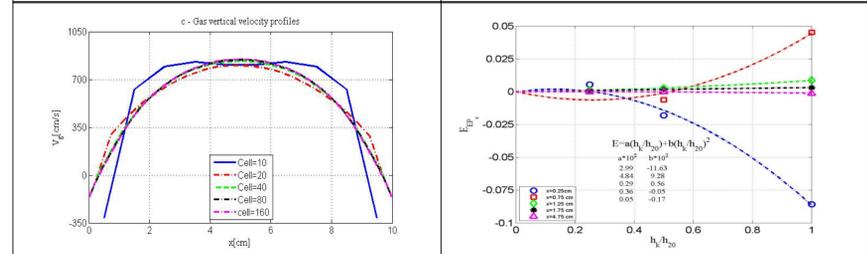


Figure 6. Time averaged quantities profiles for 10, 20, 40, 80, and 160 cells cases.

Figure 7. Grid convergence: Difference between two successive meshes at specific location, k is 80, 40 and 20 in solid phase volume fraction

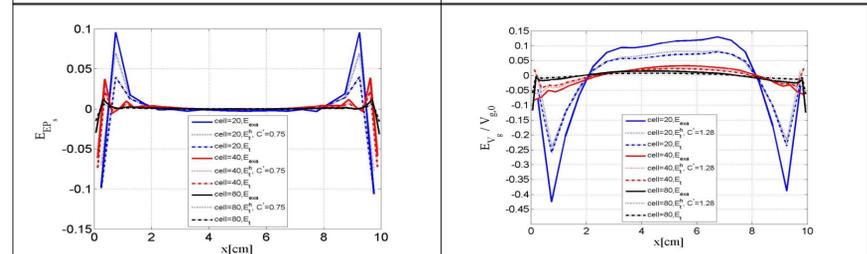


Figure 8. Error in solid-phase volume fraction

Figure 9. Error in gas phase velocity; $V_{g,0}$ is the initial gas phase velocity

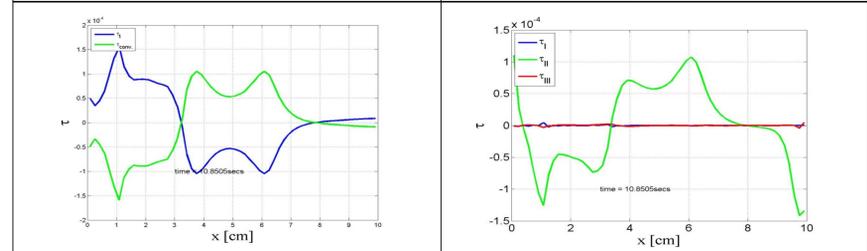


Figure 10. Time derivative and convection term contribution to τ at $t = 10.85$ sec.

Figure 11. Source terms in Eq. 6 at $t = 10.85$ sec.

Figures 10&11 show that the error source is dominated by term-II, i.e. the error in the gas-phase velocity.

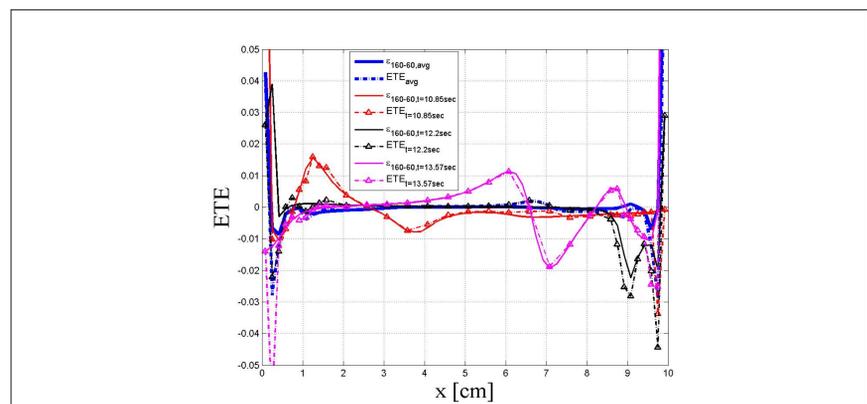


Figure 12. Time averaged and temporal errors in gas phase volume fraction, calculated by the difference between cell 160 and cell 60 cases (ϵ_{160-60}), and error transport equation (ETE).

Blue curves show the error transport equation can predict the time averaged errors accurately. Furthermore, the transients of the error can also be predicted by the ETE with reasonable accuracy.

Conclusions

- An error transport equation (ETE) is formulated for applications in conjunction with multi-phase flow codes such as MFIx to predict the discretization error field distribution.
- Verification against exact solution of Burgers equation with 'smooth' transient solutions yielded very good results on single-grid simulations.
- Application to 1D-transient/periodic core annular flow case using MFIx showed encouraging results when two simulations on two different meshes are utilized in evaluation of the error source terms in the ETE for the gas-phase volume fraction.
- Future work will focus on extending the same method to other variables and more realistic 2D transient flow cases.

Acknowledgement

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References

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